

# $W$ -boson production in spin-dependent global analysis

Pavel Nadolsky & Randall Scalise  
Southern Methodist University

- ✓ Usefulness of lepton-level asymmetries  $A_L(y_\ell)$  in  $W$  boson production
- ✓ Implementation of NLO  $A_L(y_\ell)$  in the global fits

## Next-to-leading order (NLO) corrections in the analysis of parton distributions

- ✓ Required to make accurate predictions
- ✓ Would drastically slow calculations if straightforwardly implemented in the fit

Common solution: calculate the NLO cross section as

$$\sigma_{NLO} = K \sigma_{LO},$$

where

- ✓ the LO cross section  $\sigma_{LO}$  is updated in each call of the minimization subroutine
- ✓ the more complicated factor  $K \equiv \sigma_{NLO}/\sigma_{LO}$  is updated every  $n$  calls (where  $n$  is a large number, e.g.,  $n \sim 10^3$ )

## $K$ -factors in the spin-dependent fit

In the polarized case, the convergence of such procedure is questioned due to

- ✓ flexibility and indefinite sign of spin-dependent distributions  $\Delta f(x, Q)$
- ✓ possible presence of radiation zeros ( $\sigma_{LO} = 0$ ) in spin-dependent cross sections

An alternative method involves a complete calculation of  $\sigma_{NLO}$  in each call of minimization using Mellin transform (*M. Stratmann, W. Vogelsang, Phys. Rev. D64, 114007*)

✿ I will argue that the  $K$ -factors provide an efficient way to implement NLO corrections in polarized  $W$  boson production

$\Delta pp \rightarrow (W^\pm \rightarrow l\nu)X$ :

asymmetry  $A_L(y_\ell)$  with respect to the rapidity  $y_\ell$  of the decay charged lepton (P. N., C.-P. Yuan, NPB666, 3 (2003); *ibid.*, B666, 35 (2003))

$$A_L(y_\ell) \equiv \frac{\frac{d\sigma^{p \rightarrow p}}{dy_\ell} - \frac{d\sigma^{p \leftarrow p}}{dy_\ell}}{\frac{d\sigma^{p \rightarrow p}}{dy_\ell} + \frac{d\sigma^{p \leftarrow p}}{dy_\ell}}$$

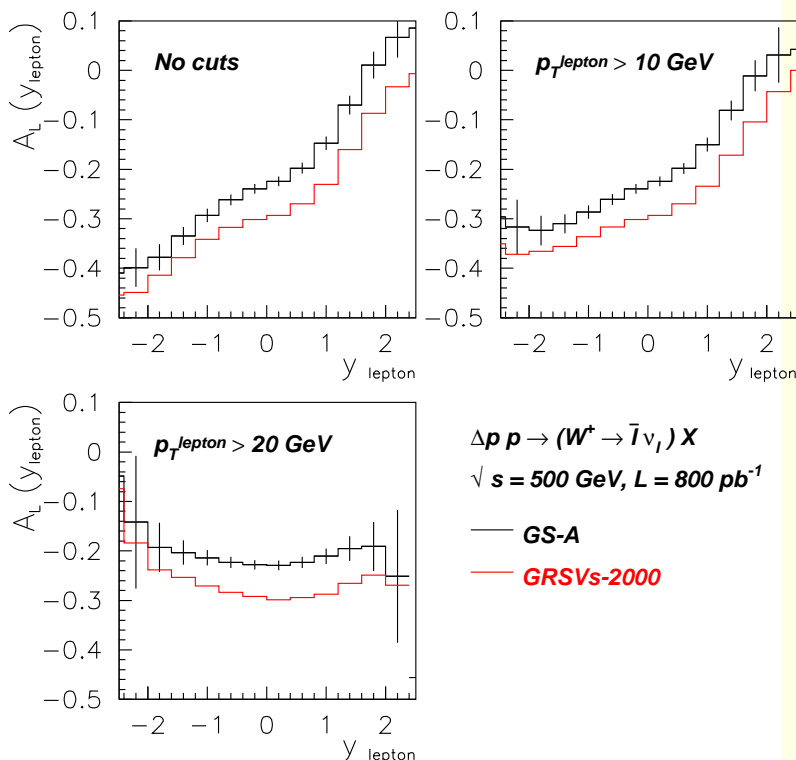
A better alternative to the commonly discussed single-spin asymmetry  $A_L(y)$  with respect to the rapidity  $y$  of the  $W$  boson

- ✓ Directly measurable
- ✓ Not distorted by limited acceptance of RHIC detectors (while  $A_L(y)$  is strongly distorted)
- ✓ Sensitive to different polarized parton distributions
- ✓ A fully differential  $\mathcal{O}(\alpha_S)$  calculation with inclusion of  $W$ -boson decay and transverse momentum resummation exists in the form of a Monte-Carlo code

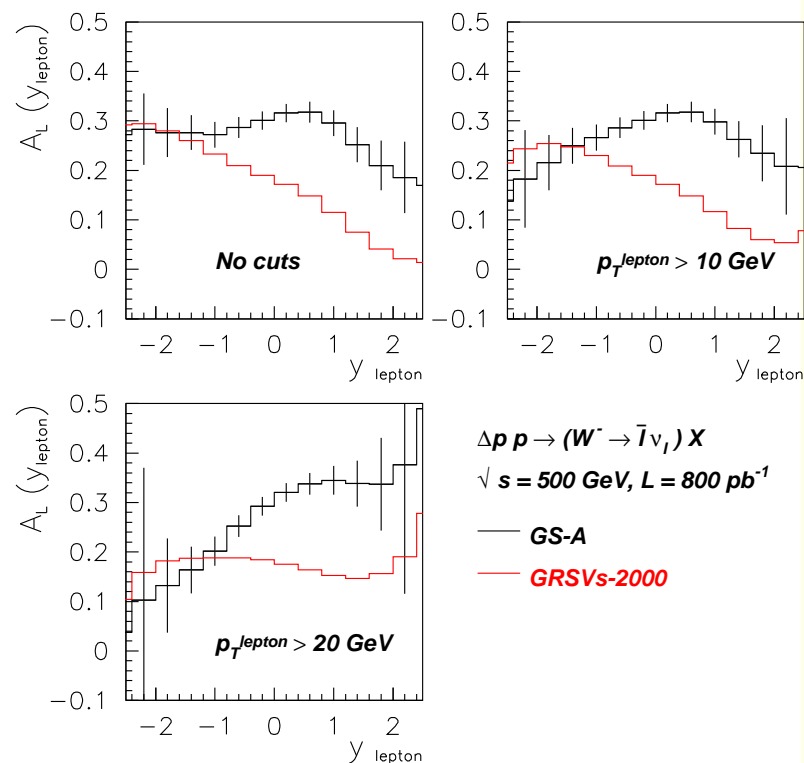
(available at <http://hep.pa.msu.edu/~nadolsky/RhicBos>)

$A_L(y_\ell)$  for different choices of  $\min p_{T\ell}$

$\Delta pp \rightarrow W^+ X$



$\Delta pp \rightarrow W^- X$



The direct experimental observable is  $A_L(y_\ell)$  with  $p_{T\ell} \geq p_{T\ell}^{\min}$

Unpolarized  $W$ -boson charge asymmetry at the Tevatron

$$A_{charge}(y_\ell) \equiv \frac{\frac{d\sigma^{W^+}}{dy_\ell} - \frac{d\sigma^{W^-}}{dy_\ell}}{\frac{d\sigma^{W^+}}{dy_\ell} + \frac{d\sigma^{W^-}}{dy_\ell}}$$

- ✓ analog of  $A_L(y_\ell)$  in the unpolarized case; related to

$$A_{charge}(y) = \frac{u(x_a)d(x_b) - d(x_a)u(x_b)}{u(x_a)d(x_b) + d(x_a)u(x_b)}$$

- ✓ constrains  $d(x, M_W)/u(x, M_W)$  in CTEQ and MRST analyses
- ✓ published data is implemented in the global fit with the selection cut  $p_{T\ell} \geq p_{T\ell}^{\min} = 25 \text{ GeV}$

$d\sigma/dy_\ell$  at the Born level

$$\left( \frac{d\sigma(p\bar{p} \rightarrow W^+ X)}{dy_\ell} \right)_{LO} = \frac{2\pi\sigma_0}{S} \int_{y_{\min}(p_{T\ell}^{\min})}^{y_{\max}(p_{T\ell}^{\min})} dy \sin^2 \theta \\ \times \left\{ u(x_a) d(x_b) (1 + \cos \theta)^2 + d(x_a) u(x_b) (1 - \cos \theta)^2 \right\},$$

with  $x_{a,b} = \frac{Q}{\sqrt{S}} e^{\pm y}$ ,  $\cos \theta = \tanh(y_\ell - y)$

- ✓ Simple kinematics due to  $p_{TW} = 0$
- ✓ Only 2 structure functions  $\propto (1 \pm \cos \theta)^2$  in the  $W^+$  rest frame
- ✓  $p_{T\ell}^{\min}$  appears only in the limits of the integration  $y_{\min}$ ,  $y_{\max}$

Similarly,

for  $d\Delta\sigma/dy_\ell$ :

$$\left( \frac{d\Delta\sigma(pp \rightarrow W^+ X)}{dy_\ell} \right)_{LO} = \frac{2\pi\sigma_0}{S} \int_{y_{\min}(p_{T\ell}^{\min})}^{y_{\max}(p_{T\ell}^{\min})} dy \sin^2 \theta \\ \times \left\{ -\Delta u(x_a) \bar{d}(x_b) (1 + \cos \theta)^2 + \Delta \bar{d}(x_a) u(x_b) (1 - \cos \theta)^2 \right\}$$

NLO calculation of  $d\sigma/dy_\ell$  is more complex

- ✓ 5 structure functions due to lepton-parton spin correlations
- ✓ complicated kinematics due to  $p_{TW} \neq 0$
- ✓ integration of  $p_T, y, Q^2, p_{T\ell}$  over experimental phase space (best done with Monte-Carlo integration)
- ✓ resummation of transverse momentum logarithms needed for  $p_T$  distributions
- ✓ is implemented in CTEQ analysis using an effective  $K$ -factor



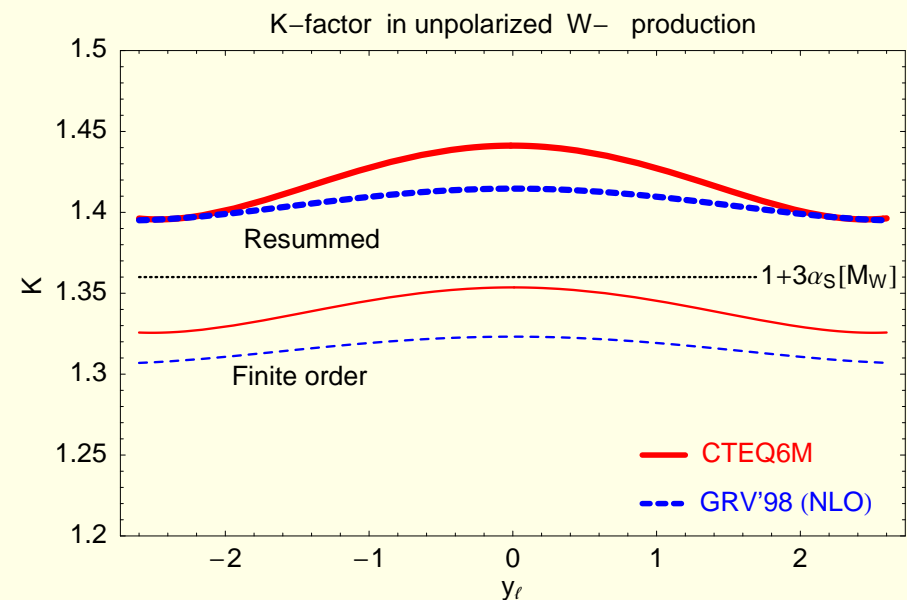
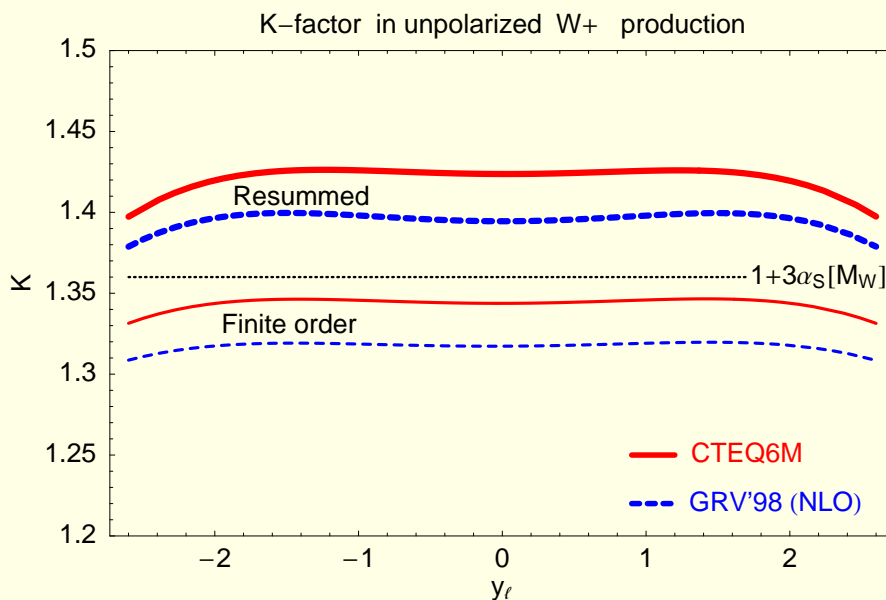
# $K$ -factor for $d\sigma/dy_\ell$ (PRELIMINARY)

(Barger & Phillips, Collider Physics, ch. 7.11)

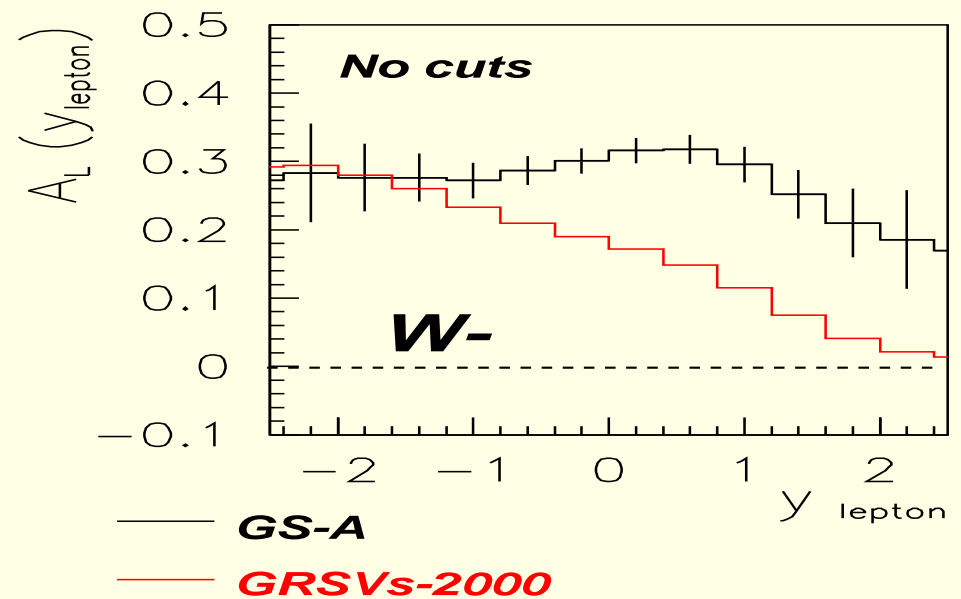
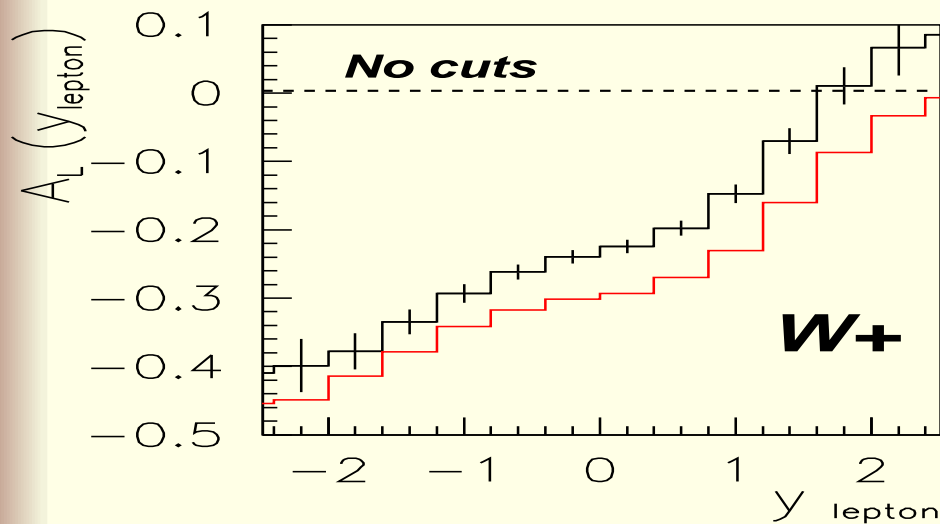
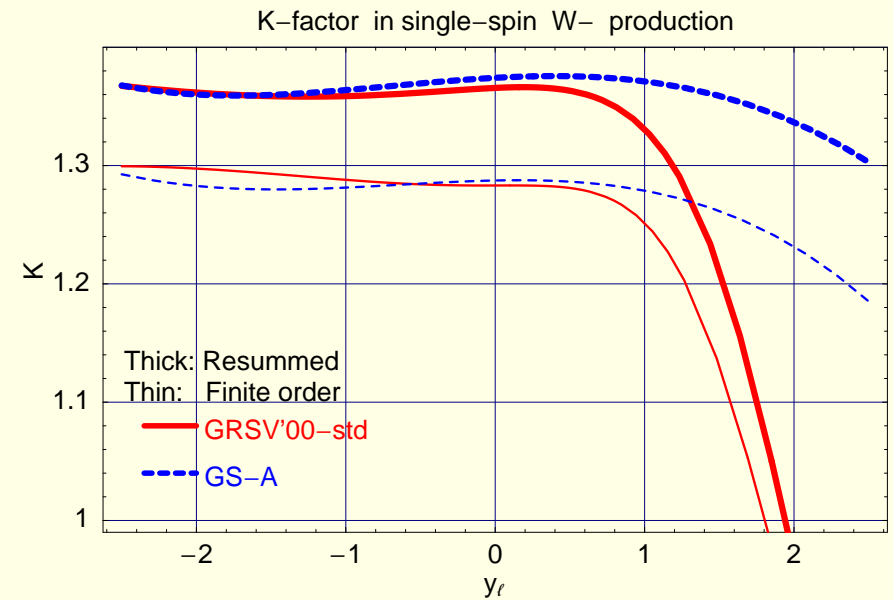
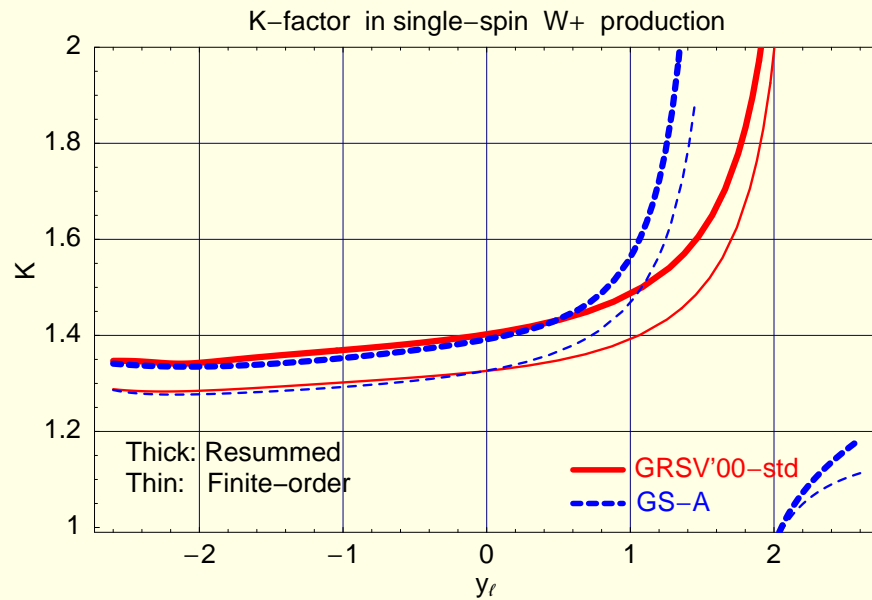
$$\frac{\frac{d(\Delta)\sigma_{NLO}}{dy_\ell}}{\frac{d(\Delta)\sigma_{LO}}{dy_\ell}} \approx \left( \underbrace{1 + 3\alpha_s(Q)}_{K_0} + \text{extra terms} \right)$$

$K_0$  is independent of the PDF and spin ( $K_0 \approx 1.36$  at  $Q = M_W$ )

The extra terms depend on the PDF and spin; they are small in the unpolarized case



# $K$ -factors for $d\Delta\sigma/dy_\ell$ (PRELIMINARY)



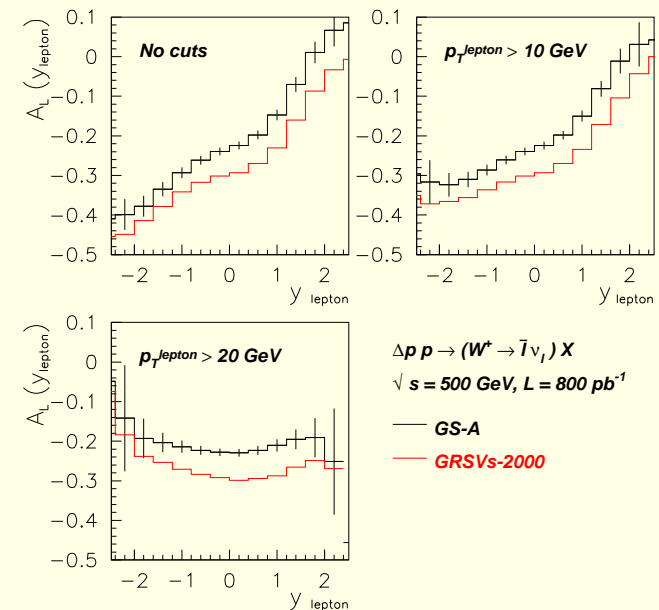
Spin-dependent  $K$ -factors diverge near the LO radiation zeros

$$\text{Radiation zero: } \left( \frac{d\Delta\sigma}{dy_\ell} \right)_{\text{LO}} = 0$$

Is that a problem?

## Radiation zeros...

- ✓ ...are easily identifiable in the data ( $|A_L(y_\ell)| \lesssim 0.1$ )
- ✓ ...are smeared by experimental resolution and statistical errors
- ✓ ...can be removed from the data by  $p_{T\ell}$  cuts
- ✓ ...can be excluded from the fit by data selection cuts
- ✓ ...can be included in the fit, with the direct NLO calculation used only in the vicinity of the radiation zero
- ✓ no radiation zero for the cut  $p_{T\ell} > 20$  GeV



## Summary

1. The lepton single-spin asymmetry  $A_L(y_\ell)$  provides a theoretically clean and direct observable in polarized  $W$ -boson production
2. Resummed  $A_L(y_\ell)$  (with  $p_{T\ell}$  cuts) can be easily implemented in the global fits using an effective  $K$  factor  $K = \sigma_{NLO}/\sigma_{LO}$  and a simple procedure for handling radiation zeros
3. The  $K$ -factors (calculated in the  $p_T$  resummation formalism) for Gehrmann-Stirling, GRSV, and de Florian-Sassot PDF sets can be generated using RhicBos (input Xsection grids available by request)

## Double Mellin transform

$$\sigma = \frac{1}{(2\pi i)^2} \int_{C_n} dn \int_{C_m} dm \Delta f_n \Delta f_m \tilde{\sigma}(m, n),$$

where

$$\Delta f_m \equiv \int_0^1 dx x^{m-1} \Delta f(x)$$

is the  $m$ -th moment of  $\Delta f(x)$ ,

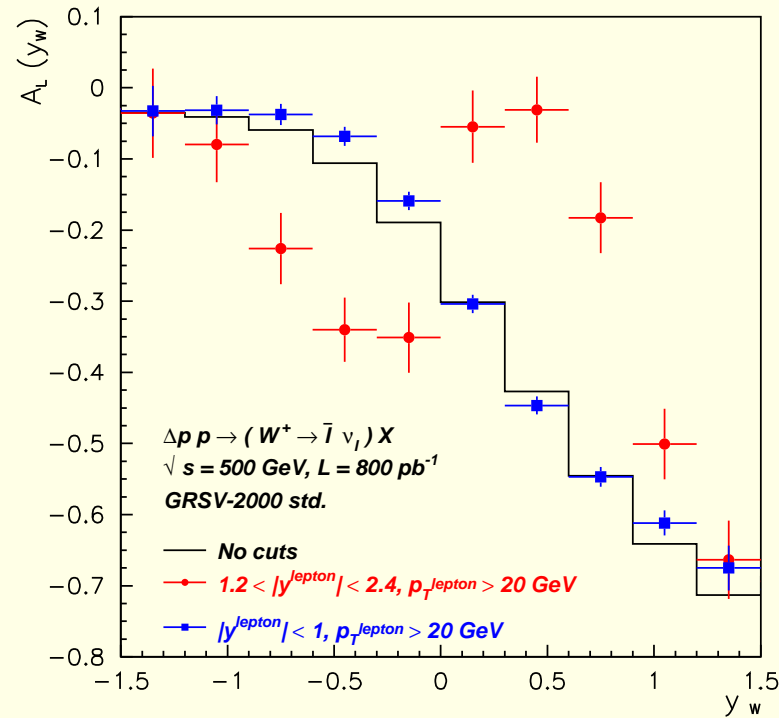
$$\tilde{\sigma}(m, n) = \int d\{P.S.\} \int_0^1 dx_a \int_0^1 dx_b x_a^{-n} x_b^{-m} \frac{d\hat{\sigma}(x_a, x_b)}{d\{P.S.\}}$$

is the convolution of the cross section  $\sigma(x_a, x_b)$  (integrated over phase space P.S.) with the “eigenvector PDFs”  $x_a^{-m}$ ,  $x_b^{-n}$

$\sigma(m, n)$  can be calculated at the full NLO before the fitting

It is not obvious that  $\int d\{P.S.\}$  can be evaluated using Monte-Carlo methods for complex  $m$  and  $n$

# Impact of leptonic cuts on the measurement of $A_L(y)$



Due to the spin-1 of  $W^\pm$  boson, cuts affect the numerator and denominator of  $A_L(y)$  differently

Interest in fully differential cross sections **at the lepton level**

✓ Partial angular coverage of Phenix and Star detectors

◇  $\cancel{E}_T$  and momentum of  $W^\pm$  **cannot** be reconstructed

◇  $y$  can be **approximately** reconstructed in a limited event **sample** and only if dynamics is well understood

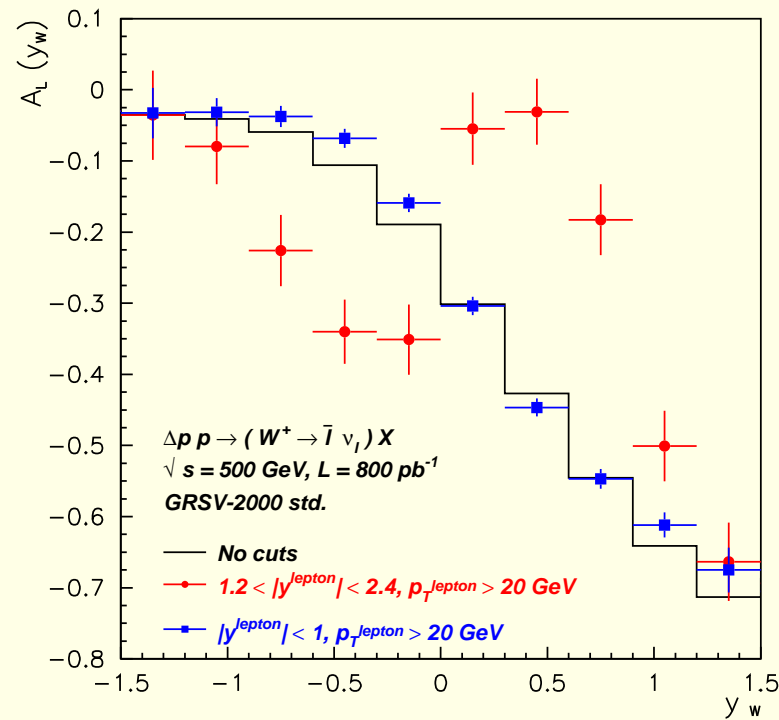
◇

$$A_L(y)|_{\substack{\text{with} \\ \text{lepton} \\ \text{cuts}}} \neq A_L(y)|_{\substack{\text{without} \\ \text{lepton} \\ \text{cuts}}}$$

Due to the spin-1 of  $W^\pm$  boson, cuts affect the numerator and denominator of  $A_L(y)$  differently

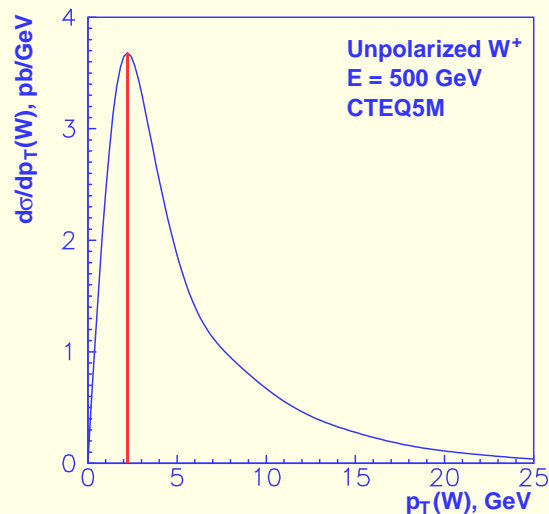


# Impact of leptonic cuts on the measurement of $A_L(y)$



Due to the spin-1 of  $W^\pm$  boson, cuts affect the numerator and denominator of  $A_L(y)$  differently

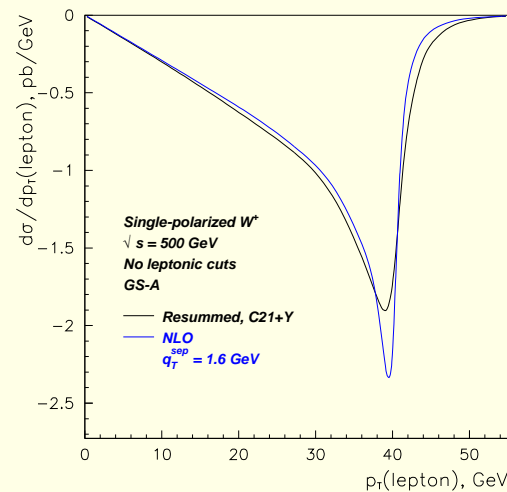
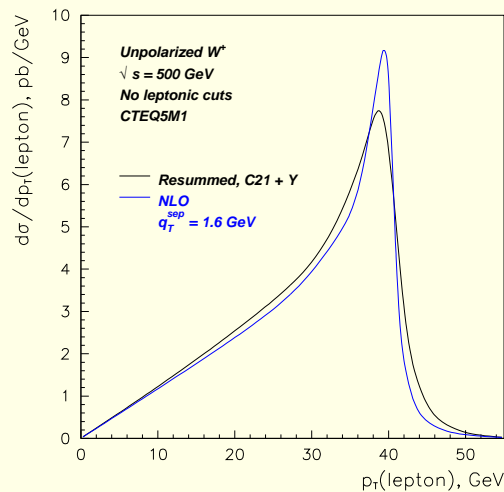
# Transverse momentum distributions



$p_{TW} \neq 0$ ! The shape of  $d\sigma/dp_{TW}$  at  $p_{TW} \rightarrow 0$  cannot be described at a finite order of PQCD: calculation of the sum

$$\frac{1}{p_{TW}^2} \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \sum_{m=0}^{2n-1} v_{mn} \left( \ln^m \frac{Q^2}{p_{TW}^2} \quad \text{or} \quad \delta(\vec{p}_{TW}) \right)$$

is needed



Similar multiple parton radiation effects in lepton  $p_T$  distributions